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Wormhole phase in the RST model

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Abstract: We show that the RST model describing the exactly soluble black hole model can have a dynamical wormhole solution along with an appropriate boundary condition. The necessary exotic matter which is usually negative energy density is remarkably produced by the quantization of the infalling matter fields. Then the asymptotic geometry in the past is two-dimensional anti-de Sitter (AdS_2), which implies the exotic matter is negative. As time goes on, the wormhole eventually evolves into the black hole and its Hawking radiation appears. The throat of the static RST wormhole is lower-bounded but in the presence of infalling matter it collapses to a black hole.

Keywords: Models of Quantum Gravity, 2D Gravity.

Contents

1. Introduction

A two-dimensional quantum gravity coupled to conformal matters has been extensively studied [1]. It is exactly soluble at the classical level, however, the semiclassical equations of motion have not been solved in a closed form. Then, the model has been modified by a number of authors to allow explicit construction for exact quantum black hole solutions. Among them, Russo, Susskind, and Thorlacius (RST) [2] have added a specific term, rather than modifying the dilaton potential as was done in refs. [3]. They have constructed an exactly solvable semiclassical model, which has semiclassical solutions describing the formation and the evaporation of black holes with an asymptotic-flat boundary condition.

On the other hand, there has been great interest in space-time wormholes [4, 5] describing travels to other universes, interstellar travels, and time machines, etc. Classically, in order to construct a lorentzian wormhole, the exotic matter violating Weak Energy Condition (WEC), Null Energy Condition (NEC), Strong Energy Condition (SEC), and/or Dominant Energy Condition (DEC), etc. is required [6, 7]. Especially, for exactly soluble wormhole models, the ghost fields have been used in obtaining stable wormhole solutions [8, 9, 10, 11]. However, if one considers the quantum theory rather than the above-mentioned classical one, then it might be more plausible to get the exotic source from the quantum regime since our universe is eventually governed by quantum mechanics.

So, in this paper, we would like to consider the energy-momentum tensors from the nonlocal term generated by the one-loop effective action in the RST model as a candidate for the exotic source, instead of the artificial classical negative energy density. In fact, it has been shown that the arbitrarily small violations of energy conditions yield a wormhole solution [12]. Then, we find the wormhole phase different from the conventional black hole phase in the RST model. Since the quantum-corrected energy-momentum tensors are sensitive to the boundary condition, we can use appropriate energy-momentum tensors satisfying the constraint equations for the wormhole geometry. Thus, the quantum-mechanically induced negative energy density produces the static wormhole, and as time goes on it eventually evolves into a black hole.

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We recast the RST model to obtain the general solution, in section 2, and present a wormhole solution by imposing the static-wormhole boundary condition at the earliest time, which yields an asymptotic-AdS geometry. In section 3, the exoticity function [5] is evaluated for our model and the asymptotic behavior of the quantum correction of the stress-energy is presented. Finally, summary and discussions are given in section 4.

2. Wormhole solution in the RST model

It has been well-known that the RST model gives evaporating black hole or minkowskian vacuum solutions depending on the boundary conditions, however, other geometries have not been considered yet. So one might think the other kind intriguing geometries, specifically, wormhole soulutions by choosing the other appropriate boundary conditions. Of course, the essential motive is to obtain the wormhole solution without the *ad hoc* classical negative energy density. We will show that the RST model naturally involves our desirable wormhole solution, which is a big difference from the previous works [8, 9, 10, 11].

We begin with the two-dimensional dilaton gravity action coupled to conformal fields with the following quantum correction term in two dimensions,

$$S = S_{DG} + S_f + S_{qt}\,, \tag{2.1}$$

$$S_{DG} = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2\right], \tag{2.2}$$

$$S_f = \frac{1}{2\pi}\int d^2x\sqrt{-g}\sum_{i=1}^{N}\left[-\frac{1}{2}(\nabla f_i)^2\right], \tag{2.3}$$

$$S_{qt} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left[-\frac{\kappa}{2}\phi R - \frac{Q^2}{2}R\frac{1}{\Box}R\right], \tag{2.4}$$

where g, ϕ, and f_i's are the metric, the dilaton field, and the conformal matter fields, and λ^2 is a cosmological constant. Choosing $\kappa = 2Q^2 = \frac{N-24}{12}$ yields the well-known RST model, where N is the number of conformal matter fields, which is taken to be large. This model gives a dynamical black hole solution with the positive Arnowitt-Deser-Misner (ADM) mass [2].

In the conformal gauge defined by $g_{+-} = -\frac{1}{2}e^{2\rho}, g_{--} = g_{++} = 0$, where $x^{\pm} = (x^0 \pm x^1)$, the above action is written as [2]

$$S = \frac{1}{\pi}\int d^2x\left[-\partial_+\chi\partial_-\chi + \partial_+\Omega\partial_-\Omega + \lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} + \frac{1}{2}\sum_i \partial_+ f_i\partial_- f_i\right] \tag{2.5}$$

with the constraints

$$\kappa t_{\pm}(x^{\pm}) = \sqrt{\kappa}\partial_{\pm}^2\chi - \partial_{\pm}\chi\partial_{\pm}\chi + \partial_{\pm}\Omega\partial_{\pm}\Omega + \frac{1}{2}\sum_i \partial_{\pm} f_i\partial_{\pm} f_i\,, \tag{2.6}$$

where the functions $t_{\pm}(x^{\pm})$ reflect the nonlocality of the conformal anomaly of the action (2.4), and we performed a field redefinition to a Liouville theory [13]: $\chi = \sqrt{\kappa}\rho -$

$\sqrt{\kappa}\phi/2 + e^{-2\phi}/\sqrt{\kappa}$ and $\Omega = \sqrt{\kappa}\phi/2 + e^{-2\phi}/\sqrt{\kappa}$, where we assumed $\kappa > 0$. Then, the equations of motion are obtained from the action (2.5),

$$2\partial_+\partial_-\chi + \frac{2}{\sqrt{\kappa}}\lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} = 0\,, \tag{2.7}$$

$$-2\partial_+\partial_-\Omega - \frac{2}{\sqrt{\kappa}}\lambda^2 e^{\frac{2}{\sqrt{\kappa}}(\chi-\Omega)} = 0\,, \tag{2.8}$$

$$\partial_+\partial_- f_i = 0\,, \tag{2.9}$$

and the key ingredient of the exact solubility is due to eqs. (2.7) and (2.8). Combining these, we obtain $\partial_+\partial_-(\chi - \Omega) = 0$, and then the residual symmetry can be fixed by choosing $\chi = \Omega$ (or $\rho = \phi$) in the Kruskal gauge. The general solutions of dilaton and conformal matter are obtained as

$$\Omega(x^+, x^-) = -\frac{\lambda^2}{\sqrt{\kappa}}x^+x^- + a_+(x^+) + a_-(x^-)\,, \tag{2.10}$$

$$f_i(x^+, x^-) = f_+^{(i)}(x^+) + f_-^{(i)}(x^-)\,, \tag{2.11}$$

where $a_\pm(x^\pm)$ are integration functions determined by the constraints (2.6) as

$$\kappa t_\pm = \sqrt{\kappa}\partial_\pm^2 a_\pm + \frac{1}{2}\sum_i \partial_\pm f_\pm^{(i)}\partial_\pm f_\pm^{(i)}. \tag{2.12}$$

Integrating the constraints (2.12), the solution is obtained as

$$\begin{aligned}\Omega(x^+, x^-) = {} & -\frac{\lambda^2}{\sqrt{\kappa}}x^+x^- + \int dx^+ \int dx^+ \left(\sqrt{\kappa}t_+ - \frac{1}{2\sqrt{\kappa}}\sum_i \partial_+ f_+^{(i)}\partial_+ f_+^{(i)}\right) + \\ & + \int dx^- \int dx^- \left(\sqrt{\kappa}t_- - \frac{1}{2\sqrt{\kappa}}\sum_i \partial_- f_-^{(i)}\partial_- f_-^{(i)}\right) + C_+x^+ + C_-x^- + D\,,\end{aligned} \tag{2.13}$$

where $C_\pm$, and D are integration constants. Formally, the apparent horizon curves are given as

$$0 = \partial_\pm\Omega = C_\pm - \frac{\lambda^2}{\sqrt{\kappa}}x_{\rm h}^\mp + \int^{x_{\rm h}^\pm} dx^\pm \left(\sqrt{\kappa}t_\pm - \frac{1}{2\sqrt{\kappa}}\sum_i \partial_\pm f_\pm^{(i)}\partial_\pm f_\pm^{(i)}\right). \tag{2.14}$$

Note that the evaporating black hole solution appears [2] within the assumption that the asymptotic geometry is described by the minkowskian, which eventually fixes the boundary functions as $t_\pm = 1/4(x^\pm)^2$.

With the exact solubility maintaining, one might think of the other kind of the geometry, especially, wormhole phase which will be eventually incorporated in this model. There exist largely two kinds of intriguing objects such as black hole and wormhole depending on the boundary conditions. The latter case requires that the exotic matter source is additionally present near the throat of the wormhole, and this exotic source of the negative energy density can be produced by choosing some different boundary conditions. Explicitly, the wormhole boundary condition means that the past and the future horizon curves are coincident with each other, which is consistent with the flaring-out condition in ref. [5]. In particular, the two horizons are coincident at $x^+ = x^-$ for the static wormhole.

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Returning to our specific model, we now consider the infalling matter source as $T^f_{++} = \frac{1}{2}\sum \partial_+ f^{(i)}_+ \partial_+ f^{(i)}_+ = A\delta(x^+ - x_0)$ and $T^f_{--} = \frac{1}{2}\sum \partial_- f^{(i)}_- \partial_- f^{(i)}_- = 0$, where A is a positive constant. If we require an appropriate boundary condition making the wormhole in the past, then the past and future horizon curves are coincident with the line $x^+ = x^-$ at $x^\pm < x_0$ such that the relation $x^+_{\rm h} = x^-_{\rm h}$ can be used. Furthermore, requiring no radiation condition along $\mathcal{I}^-_R$ ($\mathcal{I}^-_L$), similarly to the RST black holes at $x^+ > x_0$ ($x^- > x_0$) [2], we can fix $C_\pm$ and $t_\pm$ to

$$C_\pm = \lambda^2 x_0/\sqrt{\kappa}\,, \qquad t_\pm(x^\pm) = \frac{\lambda^2}{\kappa}\theta(x_0 - x^\pm)\,, \tag{2.15}$$

where $\theta(x)$ is 1 for $x > 0$ and 0 for $x < 0$. Then, the solution (2.13) is expressed by

$$\begin{aligned}\Omega = D + \frac{\lambda^2}{\sqrt{\kappa}}x_0^2 - \frac{\lambda^2}{\sqrt{\kappa}}\left(x^+ - x_0\right)\left(x^- - x_0\right) - \frac{A}{\sqrt{\kappa}}\left(x^+ - x_0\right)\theta\left(x^+ - x_0\right) + \\ + \frac{\lambda^2}{2\sqrt{\kappa}}\left(x_0 - x^+\right)^2\theta\left(x_0 - x^+\right) + \frac{\lambda^2}{2\sqrt{\kappa}}\left(x_0 - x^-\right)^2\theta\left(x_0 - x^-\right)\,,\end{aligned} \tag{2.16}$$

and the resulting horizon curves are from eq. (2.16),

$$\begin{aligned}0 &= \partial_+\Omega(x^+_{\rm h}, x^-_{\rm h}) \\ &= -\frac{\lambda^2}{\sqrt{\kappa}}\left(x^-_{\rm h} - x_0\right) - \frac{A}{\sqrt{\kappa}}\theta\left(x^+_{\rm h} - x_0\right) - \frac{\lambda^2}{\sqrt{\kappa}}\left(x_0 - x^+_{\rm h}\right)\theta\left(x_0 - x^+_{\rm h}\right)\,, \end{aligned} \tag{2.17}$$

$$\begin{aligned}0 &= \partial_-\Omega(x^+_{\rm h}, x^-_{\rm h}) \\ &= -\frac{\lambda^2}{\sqrt{\kappa}}\left(x^+_{\rm h} - x_0\right) - \frac{\lambda^2}{\sqrt{\kappa}}\left(x_0 - x^-_{\rm h}\right)\theta\left(x_0 - x^-_{\rm h}\right)\,. \end{aligned} \tag{2.18}$$

Note that the two horizons are splitted by the infalling matter, which means that future horizon among the coincident horizons for the wormhole throat is getting larger due to the infalling positive energy. Before the infalling matter, the two horizons are still coincident so that the two universes are connected through the wormhole throat. However, after infalling, the wormhole becomes unstable and it evolves into the black hole described by the future event horizon, and its curvature singularity is cloaked by the horizon. Therefore, the wormhole disappears, which is shown in figure 1.

Next, to find out the meaning of D, let us define the curvature scalar R as [2],

$$R = \frac{8e^{-2\phi}}{\Omega'}\left[\partial_+\partial_-\Omega - \frac{\Omega''}{\Omega'^2}\partial_+\Omega\partial_-\Omega\right] \tag{2.19}$$

in the Kruskal gauge, where $' = d/d\phi$. Then, the singularity curve is obtained from $\Omega' = 0$ as $\Omega(x^+_{\rm s}, x^-_{\rm s}) = \sqrt{\kappa}(1 - \ln(\kappa/4))/4$ which becomes $0 = D + \lambda^2 x_0^2/\sqrt{\kappa} - \sqrt{\kappa}(1 - \ln(\kappa/4))/4 + \lambda^2(x^+_{\rm s} - x^-_{\rm s})^2/2\sqrt{\kappa}$ in a stable wormhole geometry ($x^\pm < x^\pm_0$), where the subscript denotes the singularity. Because there should not exist any singularities in a stable wormhole geometry, the constant D is restricted by the following condition,

$$D + \frac{\lambda^2}{\sqrt{\kappa}}x_0^2 - \frac{\sqrt{\kappa}}{4}\left(1 - \ln\frac{\kappa}{4}\right) = \frac{M}{\lambda} > 0\,, \tag{2.20}$$

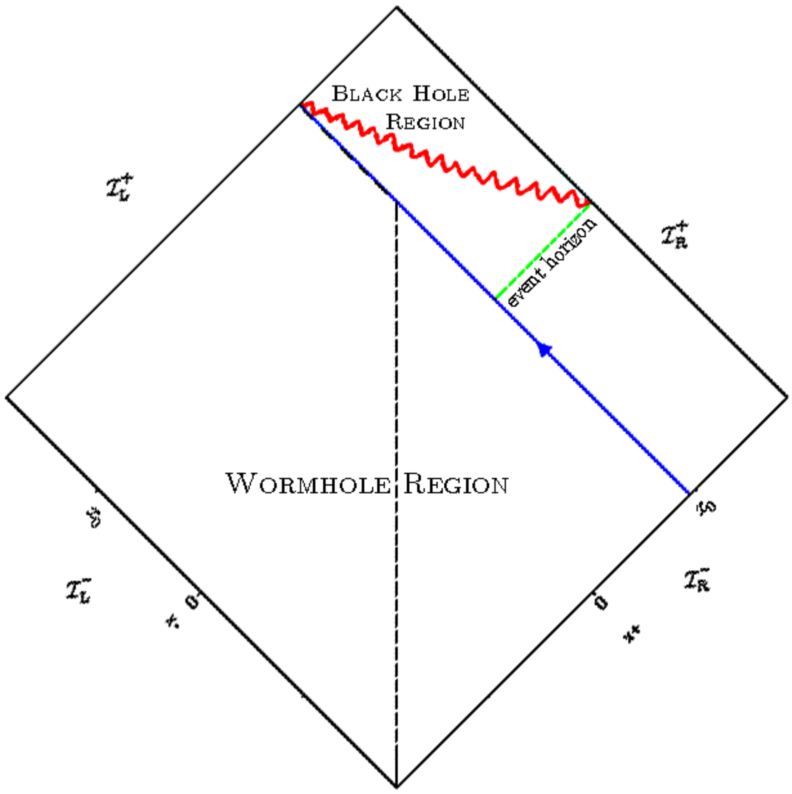

Figure 1: The Penrose diagram of the wormhole with the infalling matter source simply for $M = x_0 = \lambda = \kappa = A = 1$ shows that the conformal matter splits degenerate horizons. The thick wiggly line representing the curvature singularity is cloaked by the event horizon.

where M is a parameter related to the wormhole throat. Especially, for $M = 0$, the singularity appears along the line $x_s^+ = x_s^-$. As a result, eliminating the above D, the solution (2.16) becomes

$$\Omega = \frac{M}{\lambda} + \frac{\sqrt{\kappa}}{4}\left(1 - \ln\frac{\kappa}{4}\right) - \frac{\lambda^2}{\sqrt{\kappa}}\left(x^+ - x_0\right)\left(x^- - x_0\right) - \frac{A}{\sqrt{\kappa}}\left(x^+ - x_0\right)\theta\left(x^+ - x_0\right) + \\ + \frac{\lambda^2}{2\sqrt{\kappa}}\left(x_0 - x^+\right)^2\theta\left(x_0 - x^+\right) + \frac{\lambda^2}{2\sqrt{\kappa}}\left(x_0 - x^-\right)^2\theta\left(x_0 - x^-\right). \tag{2.21}$$

In the region of $x^\pm < x_0$, a static wormhole solution is given as

$$\Omega = \frac{M}{\lambda} + \frac{\sqrt{\kappa}}{4}\left(1 - \ln\frac{\kappa}{4}\right) + \frac{\lambda^2}{2\sqrt{\kappa}}\left(x^+ - x^-\right)^2. \tag{2.22}$$

By using this solution, we profile the curvature scalar and evaluate the size of wormhole throat. First of all, we define the dilaton field in terms of this wormhole solution through the inverse function,

$$\phi = \frac{1}{2\sqrt{\kappa}}\left[4\Omega + \sqrt{\kappa}W(\xi)\right], \tag{2.23}$$

where $W(\xi)$ satisfies $W(\xi)e^{W(\xi)} = \xi$, $\xi = -(4/\kappa)e^{-4\Omega/\sqrt{\kappa}}$, and $W(\xi) < 0$. Then, Ω' and Ω'' are obtained as $\Omega' = \frac{1}{2}\sqrt{\kappa}\left[1 + W(\xi)\right]$ and $\Omega'' = -\sqrt{\kappa}W(\xi)$, and $e^{-2\phi} = \sqrt{\kappa}\Omega''/4$. Thus, the curvature scalar (2.19) becomes

$$R = \frac{4W(\xi)}{1+W(\xi)}\left[\lambda^2 + \frac{4\lambda^4 W(\xi)}{\kappa(1+W(\xi))^2}\left(x^+ - x^-\right)^2\right]. \tag{2.24}$$

Since $-1/e \leq \xi < 0$, we naturally have two types of solutions for $W(\xi)$; one corresponds to the weak (string) coupling, $W(\xi) \leq -1$, and the other corresponds to the strong coupling, $-1 \leq W(\xi) < 0$, where the coupling is defined as $g = e^{\phi}$, and we confine the weak coupling case [1]. In the asymptotic region, $|x^+ - x^-| \to \infty$, the solution Ω diverges and ξ exponentially converges to zero, then $W(\xi) \to -\infty$. Using the following approximation, $W(\xi) \approx \ln(-\xi) \approx -2\lambda^2(x^+ - x^-)^2/\kappa$, the curvature scalar becomes $R_{\mathrm{asym}} \to -4\lambda^2$. Thus, asymptotically AdS_2 spacetime is obtained.

On the other hand, near the throat, $|x^+ - x^-| \to 0$, ξ converges as $\xi_0 = -(1/e)e^{-4M/\lambda\sqrt{\kappa}}$, then $W(\xi)$ becomes $W_0 = W(\xi_0)$. Along with the approximation of the solution Ω of $\xi = \xi_0 \exp\left[-\frac{2\lambda^2}{\kappa}\left(x^+ - x^-\right)^2\right] \approx \xi_0\left[1 - \frac{2\lambda^2}{\kappa}\left(x^+ - x^-\right)^2\right]$ in this region, the function ξ can be approximately written as

$$\begin{aligned}\xi &= W(\xi)e^{W(\xi)}\\ &\approx \xi_0 + \xi_0\frac{1+W_0}{W_0}\left[W(\xi) - W_0\right] + \xi_0\frac{2+W_0}{2W_0}\left[W(\xi) - W_0\right]^2,\end{aligned} \tag{2.25}$$

where we expanded around W_0 and used the relation $e^{W_0} = \xi_0/W_0$. For $M > 0$, $W(\xi) \approx W_0 - W_0(1 - \xi/\xi_0)/(1 + W_0)$ in the leading order of eq. (2.25), and then R is finite, $R_{\mathrm{throat}} \to 4\lambda^2 W_0/(1 + W_0)$.

As for the size of the wormhole throat, $e^{-2\phi}$ is assumed to be analogously related to higher-dimensional radial coordinate, and can be used to check whether the wormhole is closed or not, which has already been used in two-dimensional physical quantity [14]. From eq. (2.23), it is written as

$$e^{-2\phi} = -\frac{\kappa}{4}W(\xi) \equiv \lambda^2 r^2, \tag{2.26}$$

where r is a radial coordinate satisfying

$$r^2 = -\frac{\kappa}{4\lambda^2}W(\xi) \geq -\frac{\kappa}{4\lambda^2}W_0 \geq \frac{\kappa}{4\lambda^2}. \tag{2.27}$$

Then, the minimal throat radius is given as $r^2 = -\kappa W_0/4\lambda^2$. Note that it is quantum-mechanical because κ involves Plank constant.

3. Exoticity of quantum source

Now, it seems to be appropriate to mention how the result describing the wormhole geometry (2.22) comes out by directly calculating the well-known exotic condition expressed by exoticity function ζ [5]. To do so, we change the coordinate system to “the proper reference frame” (the hatted coordinate system),

$$\begin{aligned}ds^2 &= -e^{2\rho(x)}dx^+dx^-\\ &= -\exp\left(2\rho \pm \lambda\sigma^+ \pm \lambda\sigma^-\right)d\sigma^+d\sigma^-\\ &= -\left(d\sigma^{\hat{0}}\right)^2 + \left(d\sigma^{\hat{1}}\right)^2.\end{aligned}$$

where $\sigma^{\pm} = \sigma^0 \pm \sigma^1$. Then, in this coordinate system, the $(\hat{0},\hat{0})$- and $(\hat{1},\hat{1})$-components of the energy-momentum tensors $T_{\mu\nu} =< T^f_{\mu\nu} >$ are obtained from eq. (2.4) as

$$T_{\hat{0}\hat{0}} = 2\lambda^2 C_0 \left(\frac{3 - W_0}{1 + W_0}\right), \tag{3.1}$$

$$T_{\hat{1}\hat{1}} = -2\lambda^2 C_0 \,, \tag{3.2}$$

in the leading order of σ^1, near the throat, $|x^+ - x^-| \to 0$. Note that $C_0 = \exp[-4M/\lambda\sqrt{\kappa} - 1 + \ln(\kappa/4) - W_0]$, and eqs. (2.23) and (2.25) were used. So, the exoticity function reads as

$$\zeta \equiv \frac{-T_{\hat{1}\hat{1}} - T_{\hat{0}\hat{0}}}{|T_{\hat{0}\hat{0}}|} = \frac{4\lambda^2 C_0}{|T_{\hat{0}\hat{0}}|}\left(\frac{W_0 - 1}{W_0 + 1}\right) > 0\,, \tag{3.3}$$

since $W_0 < -1$. Thus, the quantum-mechanically induced energy plays the role of the exotic source since the exoticity is satisfied at the throat. In fact, *ab initio*, the classical exotic source is usually considered for lower-dimensional wormhole models [8, 9, 10, 11]. In ref. [8], the classical wormhole is given by the classical dilaton gravity action coupled to ghost fields with wrong sign kinetic term to construct wormholes. With the great help of the ghost field, the desirable classical wormhole solution is obtained, however, it seems to be awkward in that the origin of the exotic source is more or less arbitrary. In the present model, we exploited the quantum effect in order to produce the negative energy density which results in the wormhole.

Finally, we comment on the quantum radiation from the black hole, and it is explicitly calculated as $< T^f_{--} >= 2\sqrt{\kappa}\partial^2_-\Omega/(1+W(\xi)) - 4(\partial_-\Omega)^2[(1-W(\xi))/(1+W(\xi))^3] - \kappa t_-$, where eq. (2.23) is used. At the spatial infinity, $\mathcal{I}^+_R$, it is simply reduced to $< T^f_{--} >_{\mathcal{I}^+_R} \to -\lambda^2\theta(x_0 - x^-) + \kappa/4(x^- - x_0 + A/\lambda^2)^2$. Because of the first term, this might be understood by the (anti-)evaporation [15], however, it is caused by the energy-momentum tensor, $< T^f_{--} >_{\mathcal{I}^-_L} \to -\lambda^2\theta(x_0 - x^-)$ at the infinity $\mathcal{I}^-_L$. Thus, the background energy-momentum contributing to construct a wormhole is not a real radiation so that the net radiation is $< T^f_{--} >_{\mathcal{I}^+_R} - < T^f_{--} >_{\mathcal{I}^-_L} = \kappa/4(x^- - x_0 + A/\lambda^2)^2$. Note that at the past null infinities the geometry was AdS_2, which is related to the fact that the energy momentum tensor is constant, $-\lambda^2$ at $\mathcal{I}^-_L$.

4. Discussion

We have shown that the RST model has a wormhole solution along with some boundary conditions so that the wormhole can be constructed without introducing an exotic matter by hand. It is interesting to note that the throat of a static quantum wormhole (2.27) is lower-bounded when the wormhole is singularity-free. This means that it is always open while the throat of the classical wormhole can be closed [8]. In our model, the quantum wormhole collapses to the black hole, and then the infalling classical matter have been trapped in the black hole. So, it will be interesting to study how to overcome this singularity for traversability of the wormhole for further work.

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